

## REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

AD-A238 673



estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Avenue, Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

PORT DATE

3. REPORT TYPE AND DATES COVERED

Interim; 8/88 - 4/91

Dynamic Shear Band Development in Plane Strain Compression of a Bimetallic Body

5. FUNDING NUMBERS

DAAL03-88-K-0184

6. AUTHOR(S)

R. C. Batra and Z. G. Zhu

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

University of Missouri - Rolla  
Rolla, MO 65401-02498. PERFORMING ORGANIZATION  
REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

U. S. Army Research Office  
P. O. Box 12211  
Research Triangle Park, NC 27709-221110. SPONSORING/MONITORING  
AGENCY REPORT NUMBER

ARO 25396.20-MA

11. SUPPLEMENTARY NOTES

The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. **ABSTRACT** We study plane strain thermomechanical deformations of a prismatic viscoplastic body of square cross-section and deformed at a nominal strain-rate of  $5000 \text{ sec}^{-1}$ . The body has two thin layers placed symmetrically about the horizontal centroidal axis. The layer material differs from that of the body in only the value of the yield stress in a quasistatic simple compression test. The yield stress for the layer material is taken to be either one-fifth or five times that of the matrix material. Three cases, namely, when there is an elliptical void with its major axis aligned along the horizontal centroidal axis or the vertical centroidal axis, but the void center coincides with the center of the cross-section, and when there are two elliptical voids with major axes aligned along the vertical centroidal axis and a void tip abuts the layer/matrix interface are studied. The deformations are assumed to be symmetrical about the vertical and horizontal centroidal axes.

It is found that in each case shear bands initiate from points on the traction free edges where the matrix/layer interfaces intersect them and propagate into the softer material. For the soft layer these bands initially merge into one and propagate horizontally. Subsequently, each of these bands bifurcates into two which propagate into the matrix material along the direction of the maximum shear stress. There is minimal interaction between these bands and those initiating from points near the void tips.

14. SUBJECT TERMS

Shear bands, bimetallic body, thermally softening viscoplastic material

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION  
OF REPORT

UNCLASSIFIED

18. SECURITY CLASSIFICATION  
OF THIS PAGE

UNCLASSIFIED

19. SECURITY CLASSIFICATION  
OF ABSTRACT

UNCLASSIFIED

20. LIMITATION OF ABSTRACT

UL

# ARO Report 91-1

# TRANSACTIONS OF THE EIGHTH ARMY

**CONFERENCE ON APPLIED MATHEMATICS  
AND COMPUTING**



**Approved for public release; distribution unlimited.  
The findings in this report are not to be construed as  
an official Department of the Army position, unless  
so designated by other authorized documents.**

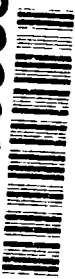
**Sponsored by**

## The Army Mathematics Steering Committee

**on behalf of**

**THE ASSISTANT SECRETARY OF THE ARMY FOR  
RESEARCH, DEVELOPMENT, AND ACQUISITION**

91-05563



# DYNAMIC SHEAR BAND DEVELOPMENT IN PLANE STRAIN COMPRESSION OF A BIMETALLIC BODY\*

R. C. Batra and Z. G. Zhu  
Department of Mechanical and Aerospace Engineering  
and Engineering Mechanics  
University of Missouri-Rolla  
Rolla, MO 65401-0249, USA

**ABSTRACT.** We study plane strain thermomechanical deformations of a prismatic viscoplastic body of square cross-section and deformed at a nominal strain-rate of  $5000 \text{ sec}^{-1}$ . The body has two thin layers placed symmetrically about the horizontal centroidal axis. The layer material differs from that of the body in only the value of the yield stress in a quasistatic simple compression test. The yield stress for the layer material is taken to be either one-fifth or five times that of the matrix material. Three cases, namely, when there is an elliptical void with its major axis aligned along the horizontal centroidal axis or the vertical centroidal axis, but the void center coincides with the center of the cross-section, and when there are two elliptical voids with major axes aligned along the vertical centroidal axis and a void tip abuts the layer/matrix interface are studied. The deformations are assumed to be symmetrical about the vertical and horizontal centroidal axes.

It is found that in each case shear bands initiate from points on the traction free edges where the matrix/layer interfaces intersect them and propagate into the softer material. For the soft layer these bands initially merge into one and propagate horizontally. Subsequently, each of these bands bifurcates into two which propagate into the matrix material along the direction of the maximum shear stress. There is minimal interaction between these bands and those initiating from points near the void tips.

**INTRODUCTION.** Adiabatic shear bands are narrow regions of intense plastic deformation that form during high strain-rate processes, such as shock loading, ballistic penetration, metal forming, and machining. As these bands generally precede material fracture, a knowledge of factors that inhibit or enhance their growth is essential to the production of durable materials and more efficient manufacturing processes. These bands form in both ferrous and nonferrous alloys.

Johnson (1987) has recently pointed out that Tresca (1878) and Massey (1921) observed hot lines, now referred to as adiabatic shear bands, during the forging of platinum. Both Tresca and Massey stated that these were the lines of greatest sliding, and also therefore the zones of greatest development of heat. Wulf (1978) has reported experimental observations of adiabatic shear bands in high strain rate ( $2000$  to  $25000 \text{ sec}^{-1}$ ) compression of 7039 aluminum armour. He found that the cross-section of the cylindrical specimens changed from circular to elliptical after the compression test, and adiabatic shear bands formed in the specimens which subsequently failed by crack propagation along the dominant band. Further references to the analytical, numerical and experimental work on shear banding may be found in two recent books (Dodd and Bai (1987), Semiatin and Jonas (1984)).

Recently, LeMonds and Needleman (1986), Needleman (1989), Anand et al. (1988), Zbib and Aifantis (1988), Batra and Liu (1989,1990), Zhu and Batra (1990), Batra and Zhu (1990), and Batra and Zhang (1990) have studied the phenomenon of shear banding in plane strain deformations of a viscoplastic solid. Whereas Needleman studied a purely mechanical problem, other works have treated a coupled thermomechanical problem. LeMonds and Needleman, Zbib and Aifantis, and Anand et al. neglected the effect of inertia forces on the ensuing deformations of the body. These investigations have employed

\* Supported by the U. S. Army Research Office Contract DAAL03-88-K-0184 to the University of Missouri - Rolla.

different constitutive relations, different techniques to integrate the stiff set of governing partial differential equations, and have generally assumed that the entire body or the portion of the body whose deformations were analyzed had only one defect in it. The prismatic body whose plane strain thermomechanical deformations are studied herein is of a square cross-section and has two thin layers made of a viscoplastic material different from that of the body and placed symmetrically about and parallel to the horizontal centroidal axis. These horizontal layers may be thought of as representing planes of chemical inhomogeneity. Also, as stated above in the abstract, there is either an elliptical void at the center of the cross-section or two ellipsoidal voids with major axes along the vertical centroidal axis and tips touching the layer/matrix interfaces. The voids can form during manufacturing. However, the symmetrical situation considered herein is to simplify the problem. The vertices of the ellipsoidal void on its major axis and the points on the free edges where the thin layer and the matrix materials meet act as nuclei for the initiation of shear bands. It thus becomes an interesting exercise to study the initiation and growth of various bands and the interaction, if any, amongst them. We account for the effect of inertia forces, strain-rate sensitivity of the materials, thermal softening effects, heat conduction, and the heat generated due to plastic working.

**FORMULATION OF THE PROBLEM.** Figure 1 depicts the cross-section of the prismatic body, and the relative dimensions of the ellipsoidal void and the two thin layers. For this case, the centers of the void and the cross-section coincide and the major axis of the void coincides with the vertical centroidal axis of the cross-section. It is assumed that the body is loaded along the vertical axis, plane strain state of deformation prevails, and that the deformations are symmetrical about the two centroidal axes. Thus the deformations of the material in the first quadrant are analyzed. We use a fixed set of rectangular cartesian coordinate axes and the referential description of motion to describe the thermomechanical deformations of the body. The governing equations are:

$$\text{balance of mass: } (\rho J)' = 0, \quad (2.1)$$

$$\text{balance of linear momentum: } \rho_0 \dot{v}_i = T_{i\alpha, \alpha}, \quad (2.2)$$

$$\text{balance of moment of momentum: } T_{i, \alpha} x_{j, \alpha} = T_{j, \alpha} x_{i, \alpha}, \quad (2.3)$$

$$\text{balance of internal energy: } \rho_0 \dot{e} = -Q_{\alpha, \alpha} + T_{i, \alpha} v_{i, \alpha}, \quad (2.4)$$

where

$$T_{i\alpha} = (\rho_0/\rho) \sigma_{ij} X_{\alpha, j}, \quad \sigma_{ij} = -\frac{B}{J} \left( \frac{\rho}{\rho_0} - 1 \right) \delta_{ij} + 2\mu D_{ij}, \quad (2.5)$$

$$2\mu = [\sigma_0/J^3 I] (1 + bI)^m (1 - \alpha\theta), \quad (2.6)$$

$$I^2 = (1/2) \bar{D}_{ij} \bar{D}_{ij}, \quad (2.7)$$

$$\bar{D}_{ij} = D_{ij} - (1/3) D_{kk} \delta_{ij}, \quad (2.8)$$

$$Q_{\alpha} = (\rho_0/\rho) q_i X_{\alpha, i}, \quad q_i = -k \theta_{, i}, \quad (2.9)$$

$$\dot{e} = c \dot{\theta} + B(\rho/\rho_0 - 1) \dot{\rho}/\rho^2. \quad (2.10)$$

In these equations  $x_i$  gives the position at time  $t$  of the material particle  $X_{\alpha}$ ,  $v_i = \dot{x}_i$  is its velocity in the  $x_i$ -direction,  $\rho$  is its present mass density,  $\rho_0$  its mass density in the reference configuration,  $J = \det [x_{i, \alpha}]$ ,  $x_{i, \alpha} = \partial x_i / \partial X_{\alpha}$ ,  $T_{i\alpha}$  is the first Piola-Kirchoff stress tensor,  $\sigma_{ij}$  is the Cauchy stress tensor,  $e$  is the specific internal energy,  $Q_{\alpha}$  is the heat flux

measured per unit area in the reference configuration,  $\underline{D}$  is the strain-rate tensor and  $\underline{\bar{D}}$  is its deviatoric part, a superimposed dot indicates material time derivative, a comma followed by index  $\alpha$  ( $j$ ) implies partial differentiation with respect to  $X_\alpha$  ( $x_j$ ), and a repeated index implies summation over the range (1,2) of the index. In the constitutive relations (2.5), (2.9) and (2.10), the material parameter  $B$  represents the bulk modulus,  $\sigma_0$  is the yield stress in a quasistatic simple compression test, parameters  $b$  and  $m$  characterize the strain-rate sensitivity of the material,  $\alpha$  describes its thermal softening,  $\theta$  equals the temperature change of a material particle from that in the reference configuration,  $k$  is the constant thermal conductivity and  $c$  is the constant specific heat. Here we have not considered the stresses caused by the thermal expansion.

The foregoing equations hold in regions occupied by the matrix and the layers. The values of material parameters for the matrix and the layer materials are the same except that either

$$\sigma_0 \text{ layer} = 5 \sigma_0 \text{ matrix}, \quad (2.11a)$$

or

$$\sigma_0 \text{ layer} = (1/5) \sigma_0 \text{ matrix}. \quad (2.11b)$$

In terms of the deviatoric stress  $\underline{s}$  defined by

$$\underline{s} = \underline{\sigma} + [B(\rho/\rho_0 - 1) - (2\mu/3) \text{tr } \underline{D}] \underline{1}, \quad (2.12a)$$

$$\underline{s} = 2\mu \underline{\bar{D}}, \quad (2.12b)$$

equations (2.12), (2.5) and (2.6) give

$$(\frac{1}{2} \text{tr } \underline{s}^2)^{1/2} = (\sigma_0/\sqrt{3}) (1 - \alpha\theta) (1 + bI)^m. \quad (2.13)$$

We assume that the body is initially at rest at a uniform temperature, has a constant mass density and is initially stress free. That is

$$\rho(\underline{x}, 0) = 1, \quad \underline{v}(\underline{x}, 0) = \underline{0}, \quad \theta(\underline{x}, 0) = 0. \quad (2.14)$$

For the material in the first quadrant, we impose the following boundary conditions.

$$\underline{v}_2 = -h(t), \quad T_{12} = 0 \text{ and } Q_2 = 0, \quad \text{on the top surface AB}, \quad (2.15)$$

$$T_{11} = 0, \quad T_{21} = 0 \text{ and } Q_1 = 0, \quad \text{on the right surface BC}, \quad (2.16)$$

$$\underline{v}_2 = 0, \quad T_{12} = 0 \text{ and } Q_2 = 0, \quad \text{on the bottom surface CD}, \quad (2.17)$$

$$T_{i\alpha} N_\alpha = 0, \quad Q_\alpha N_\alpha = 0, \quad \text{on the void surface DE}, \quad (2.18)$$

$$\underline{v}_1 = 0, \quad T_{21} = 0 \text{ and } Q_1 = 0, \quad \text{on the left surface EA}. \quad (2.19)$$

That is the top surface is moving downward with a speed  $h(t)$ , contact between it and the loading device is smooth, the right surface is traction free, and the entire boundary is ther-

mally insulated. If at any time during the deformations of the body, a point on the void surface touches the vertical axis, the boundary condition on it is changed to (2.19). The boundary conditions (2.17) and (2.18) reflect the presumed symmetry of deformations about the  $x_1$  and  $x_2$  axes. For the loading function  $h(t)$  we take

$$\begin{aligned} h(t) &= v_0 t/t_r, & 0 \leq t \leq t_r, \\ &= v_0 & t > t_r. \end{aligned} \quad (2.20)$$

The steady speed  $v_0$  of the top surface of the block is reached in time  $t_r$ .

The matrix and the layer are assumed to be perfectly bonded. Thus at the common interfaces between them, the velocity field, surface tractions, the temperature and the normal component of the heat flux are assumed to be continuous.

For other configurations of the voids, the boundary conditions are appropriately modified.

**RESULTS.** The finite element code developed by Batra and Liu (1989, 1990) was modified to analyze the present problem. In order to compute results, we used the following values of various material and geometric parameters.

$$\begin{aligned} b &= 10000 \text{ sec}, \quad \sigma_0 = 333 \text{ MPa}, \quad k = 49.22 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}, \\ m &= 0.025, \quad c = 473 \text{ J Kg}^{-1} \text{ }^\circ\text{C}^{-1}, \quad \rho_0 = 7860 \text{ Kg m}^{-3}, \\ B &= 128 \text{ GPa}, \quad H = 5 \text{ mm}, \quad v_0 = 25 \text{ m sec}^{-1}, \\ \alpha &= 0.0025 \text{ }^\circ\text{C}^{-1}. \end{aligned} \quad (3.1)$$

Thus the average applied strain-rate  $\dot{\gamma}_{\text{avg}}$  equals  $5000 \text{ sec}^{-1}$ , the reference temperature

$\theta_0 = \sigma_0/(\rho_0 c) = 89.6^\circ \text{C}$ , and  $\nu = \rho_0 v_0^2 / \sigma_0 = 0.015$ . The nondimensional number  $\nu$  signifies the effect of inertia forces relative to the flow stress of the material. For the simple shearing problem, Batra (1988) noted that the inertia forces play a noticeable role when  $\nu = 0.004$ . Thus for the present problem, the inertia forces will very likely play a significant role.

**LAYER MATERIAL SOFTER THAN THE MATRIX MATERIAL.** Figure 2 depicts the contours of the maximum principal logarithmic strain  $\epsilon$  at an average strain of 0.079 when the center of the ellipsoidal void coincides with the center of the cross-section and the major axis of the void is aligned along the vertical centroidal axis. These contours and other results reported by Zhu and Batra (1990b) reveal that shear bands initiate from points on the right traction free edge where the matrix /layer interfaces intersect it. Because the layer material is softer and its thickness quite small, these bands merge into one band that initially propagates horizontally into the layer. When the matrix material has softened somewhat due to the rise in its temperature, the horizontally propagating band bifurcates into two bands that propagate into the matrix along  $\pm 45^\circ$  directions. The band propagating into the matrix material above the layer has more severe deformations associated with it than the one propagating into the matrix material below the layer.

The band initiating from a point near the void tip on the vertical centroidal axis propagates along a line that makes an angle of  $45^\circ$  with the horizontal. This band seems to pass through the soft layer rather easily.

Figure 3 shows the contours of the maximum principal logarithmic strain  $\epsilon$  at an average strain of 0.0333 when the ellipsoidal void is at the center of the cross-section and the major axis of the void is aligned along the horizontal centroidal axis. These plots look quite similar to that for the case when the major axis of the void coincides with the vertical centroidal axis. For a further discussion and details of results in this case, see Batra and Zhu (1991).

When the void tip touches the matrix /layer interface and the major axis of the void coincides with the vertical centroidal axis, the contours of  $\epsilon$  plotted in Figure 4 at an average strain rate of 0.0175 look quite different from the previous two cases. Results given by

Batra and Zhu (1990) and these contours of  $\epsilon$  suggest that a shear band initiates within the matrix surrounding the void tip near the matrix/layer interface and propagates into the matrix material below the common interface, the direction of propagation being nearly  $45^\circ$  to the vertical axis. The shear bands initiating at points of intersection of the matrix/layer interfaces with the right traction free surface propagate into the soft layer and then bifurcate into the matrix material along lines making an angle of approximately  $45^\circ$  with the vertical. The band in the layer near the upper matrix/layer interface bifurcates into the matrix prior to that near the lower interface. Also the band in the layer near the upper matrix/layer interface continues to propagate horizontally into the layer too, while that near the lower surface does not.

**LAYER MATERIAL STRONGER THAN THE MATRIX MATERIAL.** We first study the case when the center of the ellipsoidal void coincides with that of the cross-section. In Figure 5, we have plotted contours of  $\epsilon$  at an average strain of 0.122 when the major axis of the void is vertical. Now the bands initiating from points on the right traction free edge where the matrix/layer interfaces intersect it propagate into the matrix material along lines making an angle of approximately  $\pm 45^\circ$  with the horizontal. Recall that the matrix material is softer than the layer material. The quarter of the square cross-section studied is divided into five subregions, each of which is deforming essentially rigidly, and there is a shear band at the four common boundaries (e.g., see Zhu and Batra (1990b)). However, when the major axis of the void is horizontal, contours of  $\epsilon$  depicted in Figure 6 at an average strain of 0.031 suggest a picture different from the one when the major axis of the void was vertical. We should add that the average strains in the two cases are quite different. Hence, a direct comparison is not very meaningful.

Figure 7 shows contours of  $\epsilon$  at an average strain of 0.057 when a void tip is at the matrix/layer interface. These contours and other results given by Batra and Zhu (1990) reveal that a shear band initiating from the void tip abutting the matrix/layer interface propagates initially along the interface and then into the matrix material along a line making an angle of nearly  $45^\circ$  with the vertical. The shear band initiating from the lower void tip also propagates into the matrix material along a line making an angle of approximately  $45^\circ$  with the vertical. Two shear bands also initiate from points on the right traction free edge where matrix/layer interfaces intersect it, and these bands propagate into the matrix material along lines making an angle of  $45^\circ$  with the vertical. Even though it seems that near the vertical centroidal axis a shear band has propagated into the layer, there is no localization of deformation occurring in the layer material. This is evidenced by the plots of  $\epsilon$  versus the average strain at several points in the layer that are given in Fig. 8c of Batra and Zhu's (1990) paper. The contours of the temperature rise, not included herein, support the picture laid out above for the development of four bands, two from void tips and two from points on the right traction free surface where the layer and the matrix materials meet.

**CONCLUSIONS.** We have analyzed the problem of the initiation and growth of shear bands in a prismatic viscoplastic body containing an ellipsoidal void, and two thin layers made of a different viscoplastic material placed symmetrically about the horizontal centroidal axis. The body is deformed in plane strain compression along the vertical axis at an average strain-rate of  $5000 \text{ sec}^{-1}$ , and its deformations are assumed to be symmetrical about the two centroidal axes.

Two shear bands initiate from points on the right traction free edge where the matrix/layer interfaces meet it. These bands propagate into the softer material. When the matrix material is softer, these bands propagate along lines that make an angle of approximately  $\pm 45^\circ$  with the horizontal. However, when the layer material is softer, these bands essentially merge into one and initially propagate horizontally into the layer. Subsequently, this band bifurcates into two bands that propagate into the matrix material along  $\pm 45^\circ$  directions.

Shear bands also initiate from points near the void tips and propagate in the  $\pm 45^\circ$  directions. When the layer material is stronger than the matrix material, these bands do not pass through the layer and are deflected back upon arriving at the matrix/layer interface.

However, they pass through easily through a softer layer.

## REFERENCES

- Anand, L., Lush, A. M., and Kim, K. H., 1988, "Thermal Aspects of Shear Localization in Viscoplastic Solids", Thermal Aspects in Manufacturing, M. H. Attia and L. Kops., eds., ASME - PED Vol. 30, pp. 89-103.
- Batra, R. C., and Kim, C. H., 1990, "Effect of Viscoplastic Flow Rules on the Initiation and Growth of Shear Bands at High Strain Rates", J. Mechs. Phys. Solids (in press).
- Batra, R. C., and Liu, D. S., 1990, "Adiabatic Shear Banding in Dynamic Plane Strain Compression of a Viscoplastic Material", Int. J. Plasticity, Vol. 6, pp. 231-246.
- Batra, R. C., and Liu, D. S., 1989, "Adiabatic Shear Banding in Plane Strain Problems", ASME J. Appl. Mechs., Vol. 56, pp. 527-534.
- Batra, R. C., and Zhang, X. T., 1990, "Shear Band Development in Dynamic Loading of a Viscoplastic Cylinder Containing Two Voids", Acta Mechanica (in press).
- Batra, R. C., and Zhu, Z. G., 1990, "Dynamic Shear Band Development in a Thermally Softening Bimetallic Body Containing Two Voids", Acta Mechanica (in press).
- Batra, R. C. and Zhu, Z. G., 1991, "Dynamic Adiabatic Shear Band Development in a Bimetallic Body Containing a Void", Int. J. Solids Structures (in press).
- Johnson, W., 1987, "Henri Tresca as the Originator of Adiabatic Heat Lines", Int. J. Mech. Sci., Vol. 29, pp. 301-310.
- Le Monds, J., and Needleman, A., 1986, "An Analysis of Shear Band Development Incorporating Heat Conduction", Mech. Mat., Vol. 5, pp. 363-373.
- LeMonds, J., and Needleman, A., 1986, "Finite Element Analyses of Shear Localization in Rate and Temperature Dependent Solids", Mech. Mat., Vol. 5, pp. 339-361.
- Massey, H. F., 1921, "The Flow of Metal During Forging", Proc. Manchester Assoc. Engineers, pp. 21-26.
- Needleman, A., 1989, "Dynamic Shear Band Development in Plane Strain", ASME J. Appl. Mechs., Vol. 59, pp. 1-9.
- Semiati, S. L., and Jonas, J. J., 1984, Formability and Workability of Metals: Plastic Instability and Flow Localization, ASM, Metals Park.
- Tresca, H., 1878, "On further Application of the Flow of Solids", Proc. Inst. Mech. Engr., Vol. 30, pp. 301-345.
- Wulf, G. L., 1978, "The High Strain Rate Compression of 7039 Aluminum", Int. J. Mech., Vol. 20, pp. 609-615.
- Zhu, Z. G., and Batra, R. C., 1990a, "Dynamic Shear Band Development in Plane Strain Compression of a Viscoplastic Body Containing a Rigid Inclusion", Acta Mechanica (in press).
- Zhu, Z. G., and Batra, R. C., 1990b, "Analysis of Shear Banding in Plane Strain Compression of a Bimetallic Thermally Softening Viscoplastic Body Containing an Elliptical Void", in Thermal Problems in Elasticity and Plasticity (V. Birman and D. Hui, eds.), ASME Press (in press).



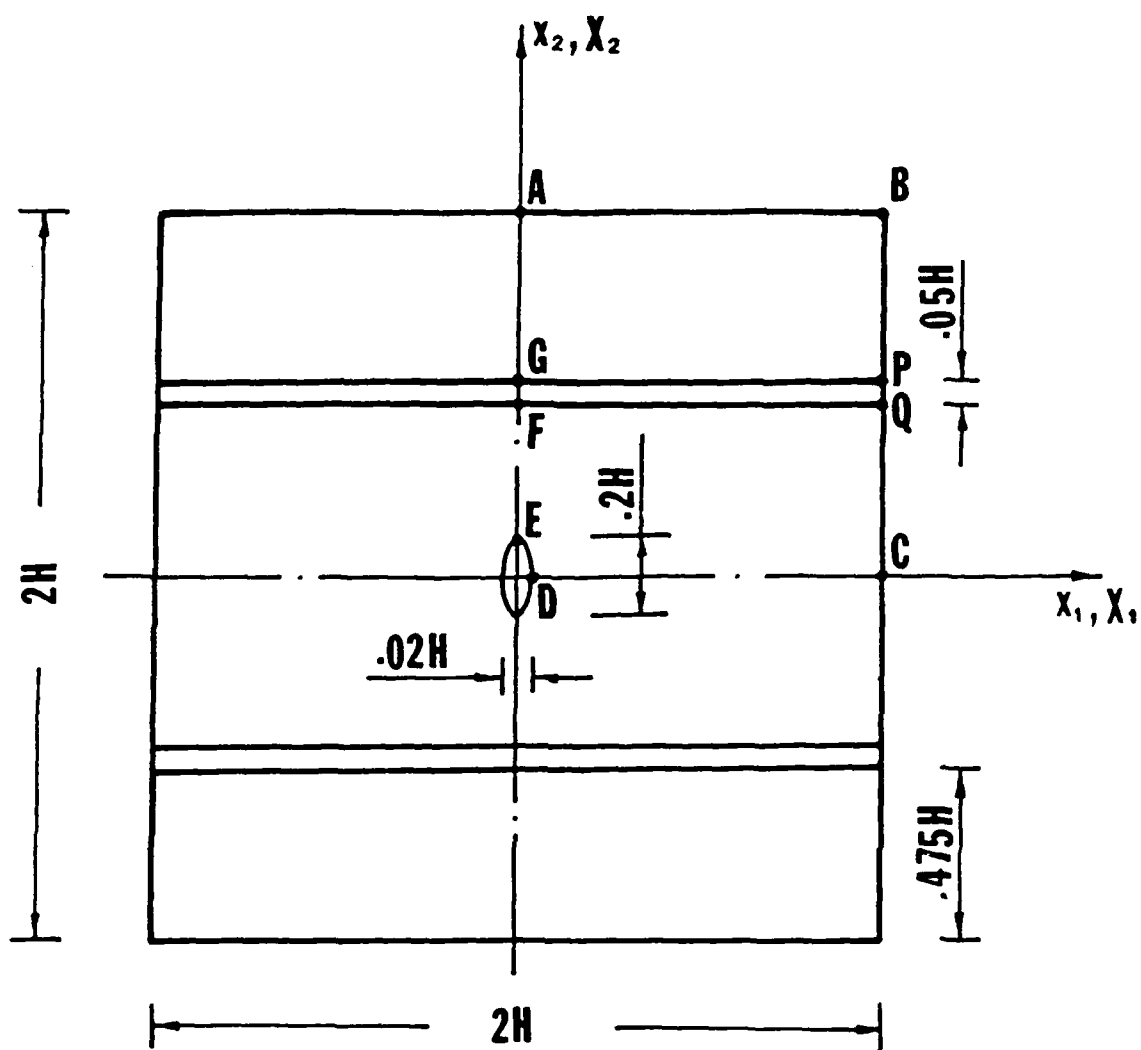


Fig. 1. Cross-section of the prismatic body studied

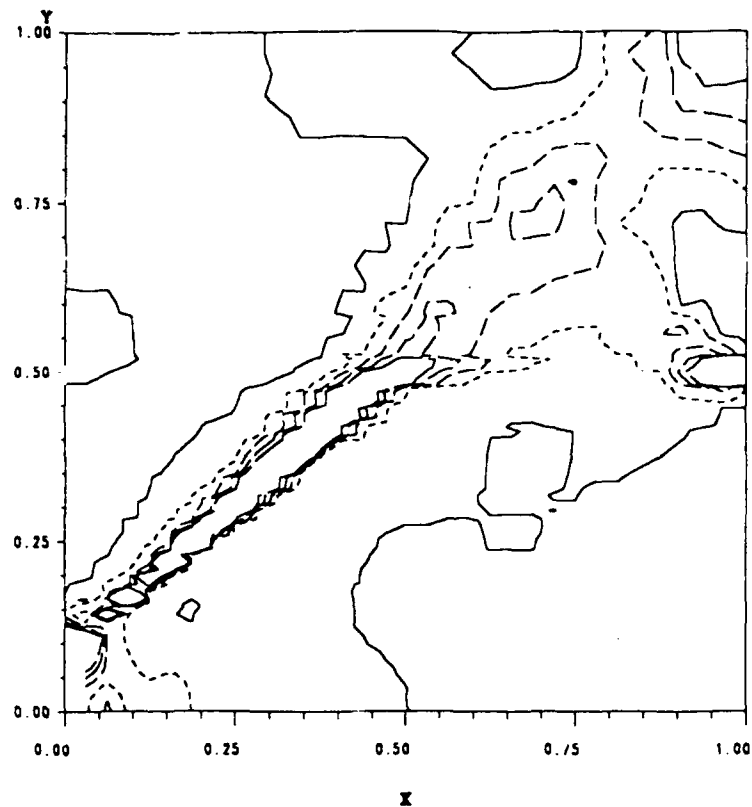


Fig. 2. Contours of the maximum principal logarithmic strain at  $\gamma_{avg} = 0.079$ .  $\sigma_o$  layer =  $(1/5) \sigma_o$  matrix. The major axis of the ellipsoidal void cc coincides with the vertical centroidal axis.  
 — 0.05, ..... 0.10, ---- 0.15, ———— 0.20, — · — · — 0.25

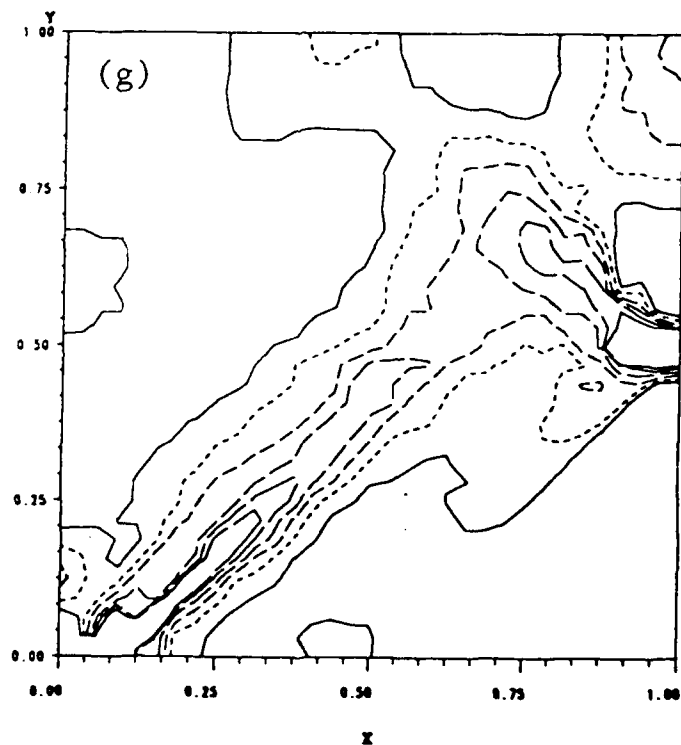


Fig. 3. Contours of the maximum principal logarithmic strain at  $\gamma_{avg} = 0.0333$ .  $\sigma_o$  layer =  $(1/5) \sigma_o$  matrix, the major axis of the ellipsoidal void coincides with the horizontal centroidal axis.  
 — 0.025, ..... 0.035, ---- 0.045, ———— 0.055, — · — · — 0.065.

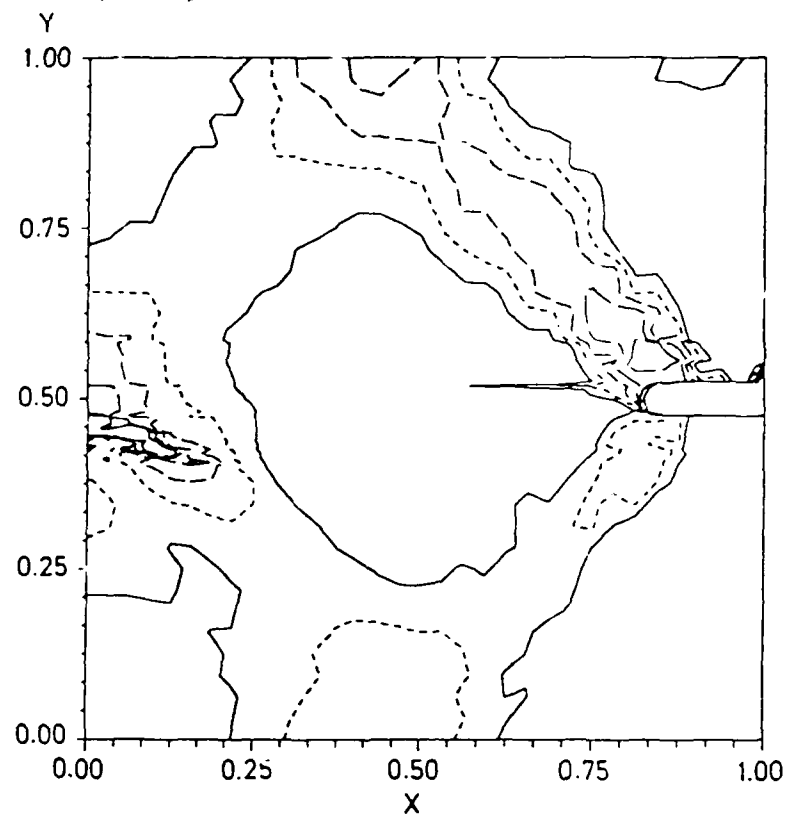


Fig. 4. Contours of the maximum principal logarithmic strain at  $\gamma_{avg} = 0.0175$ .  $\sigma_o^{layer} = (1/5) \sigma_o^{matrix}$ . A void tip is at the matrix/layer interface.  
 — 0.015, ..... 0.20, --- 0.025, — — — — 0.030, — · — · — 0.035.

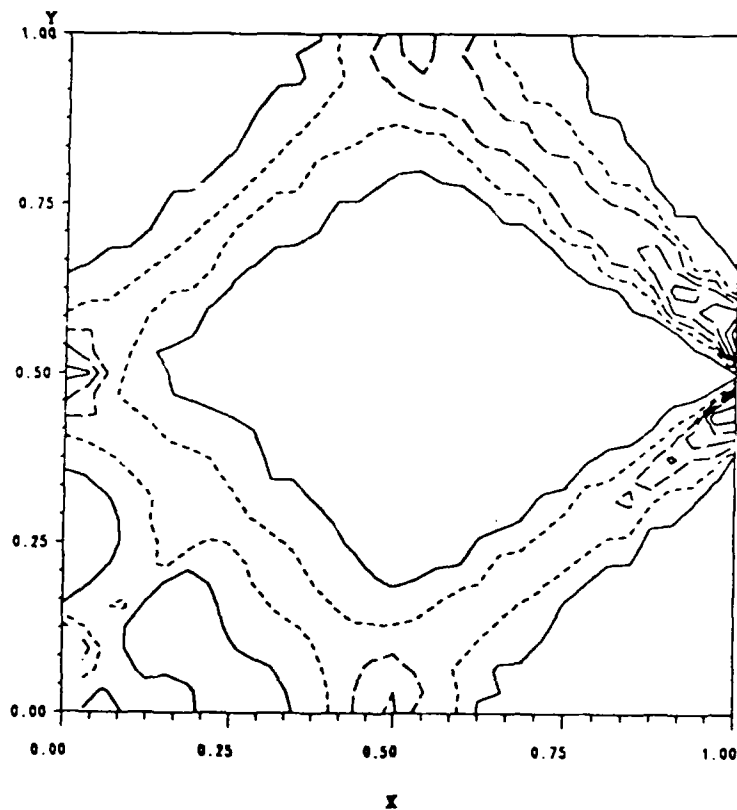


Fig. 5. Contours of the maximum principal logarithmic strain at  $\gamma_{avg} = 0.122$ .  $\sigma_o^{layer} = 5 \sigma_o^{matrix}$ . The major axis of the ellipsoidal void coincides with the vertical centroidal axis.  
 — 0.1, ..... 0.2, --- 0.3, — — — — 0.4, — · — · — 0.5.

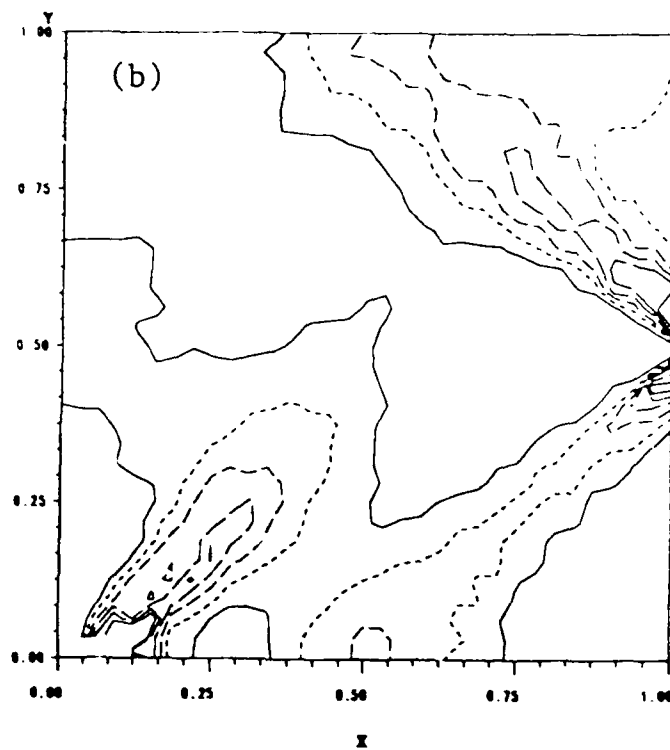


Fig. 6. Contours of the maximum principal logarithmic strain at  $\gamma_{avg} = 0.0308$ .  $\sigma_{o, layer} = 5 \sigma_{o, matrix}$ , the major axis of the ellipsoidal void coincides with the horizontal centroidal axis.  
 — 0.025, ..... 0.035, ---- 0.045, — — — — 0.055, — · — · — 0.065.

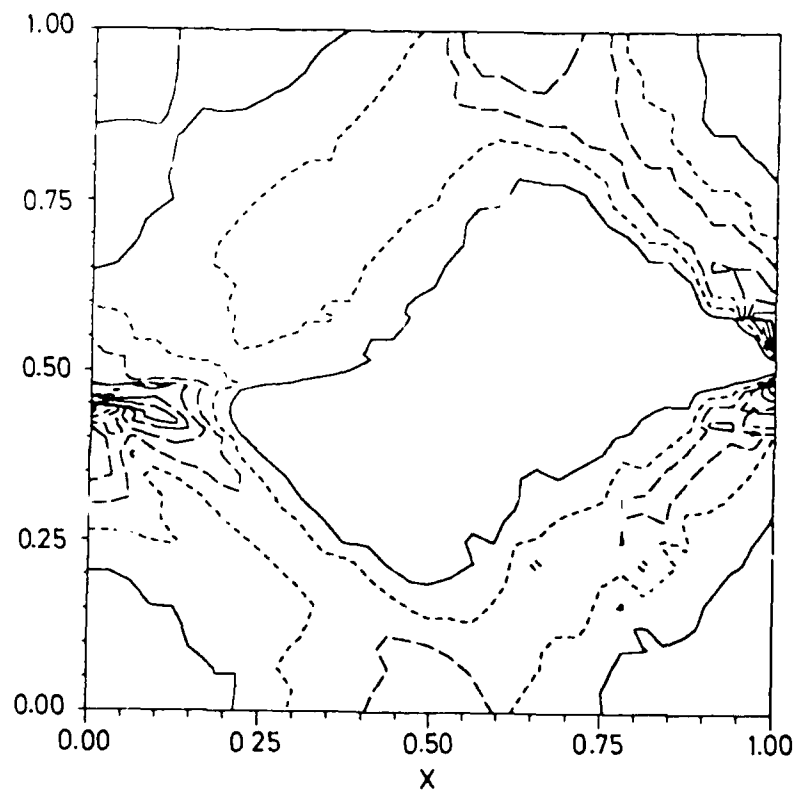


Fig. 7. Contours of the maximum principal logarithmic strain at  $\gamma_{avg} = 0.057$ .  $\sigma_{o, layer} = 5 \sigma_{o, matrix}$ . A void tip is at the matrix/layer interface.  
 — 0.05, ..... 0.06, ---- 0.07, — — — — 0.08, — · — · — 0.09.